Marwari college Darbhanga

Subject---physics Hons

Class--- B.Sc. part 2

Paper –06 ; Group—A

Topic--- Bandwidth of series LCR circuit (Electricity)

Lecture series --70

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Bandwidth of series LCR circuit

If the series RLC circuit is driven by a variable frequency at a constant voltage, then the magnitude of the current, I is proportional to the impedance, Z, therefore at resonance the power absorbed by the circuit must be at its maximum value as $P = I^2Z$.

Half-Power points

If we now reduce or increase the frequency until the average power absorbed by the resistor in the series resonance circuit is half that of its maximum value at resonance, we produce two frequency points called the half-power points which are -3dB down from maximum, taking 0dB as the maximum current reference.

Lower Cut – Off Frequency

These -3dB points give us a current value that is 70.7% of its maximum resonant value which is defined as: $0.5(I^2 R) = (0.707 \times I)^2 R$. Then the point corresponding to the lower frequency at half the power is called the "lower cut-off frequency",

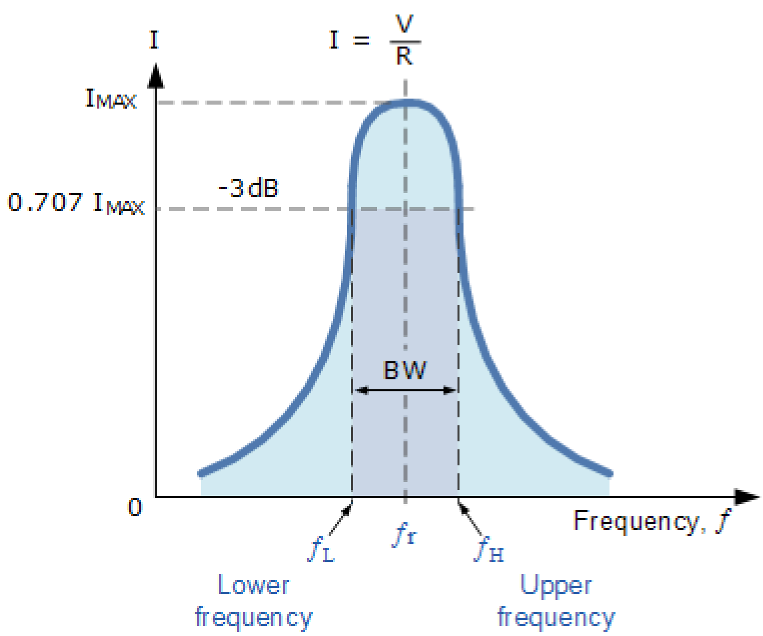
Upper Cut off Frequency

labelled f_L with the point corresponding to the upper frequency at half power being called the "upper cut-off frequency", labelled f_H.

Bandwidth

The distance between these two points, i.e. ($f_H - f_L$) is called the Bandwidth, (BW) and is the range of frequencies over which at least half of the maximum power and current is provided as shown.

Bandwidth of a Series Resonance Circuit

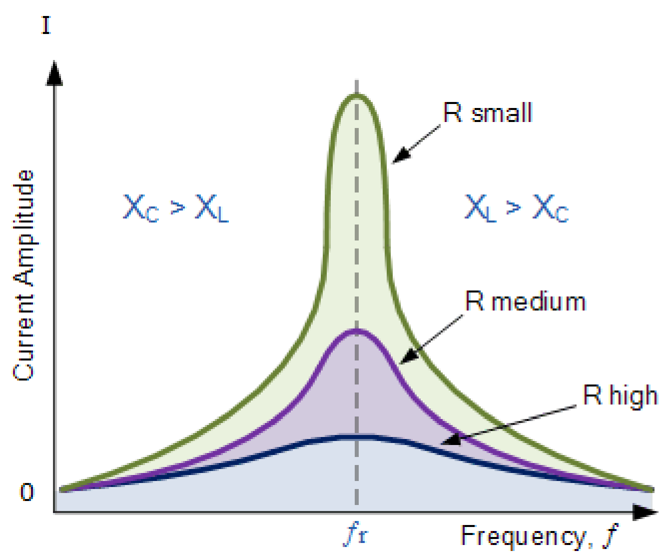


Quality Factor

The frequency response of the circuits current magnitude above, relates to the “sharpness” of the resonance in a series resonance circuit. The sharpness of the peak is measured quantitatively and is called the Quality factor, Q of the circuit. The quality factor relates the maximum or peak energy stored in the circuit (the reactance) to the energy dissipated (the resistance) during each cycle of oscillation meaning that it is a ratio of resonant frequency to bandwidth and the higher the circuit Q, the smaller the bandwidth, $Q = f_r / BW$.

As the bandwidth is taken between the two -3dB points, the selectivity of the circuit is a measure of its ability to reject any frequencies either side of these points. A more selective circuit will have a narrower bandwidth whereas a less selective circuit will have a wider bandwidth. The selectivity of a series resonance circuit can be controlled by adjusting the value of the resistance only, keeping all the other components the same, since $Q = (X_L \text{ or } X_C)/R$.

Bandwidth of a Series RLC Resonance Circuit



Then the relationship between resonance, bandwidth, selectivity and quality factor for a series resonance circuit being defined as:

1). Resonant Frequency, (f_r)

$$X_L = X_C \quad \Rightarrow \quad \omega_r L - \frac{1}{\omega_r C} = 0$$

$$\omega_r^2 = \frac{1}{LC} \quad \therefore \quad \omega_r = \frac{1}{\sqrt{LC}}$$

2). Current, (I)

at ω_r $\quad Z_T = \text{min}, \quad I_S = \text{max}$

$$I_{max} = \frac{V_{max}}{Z} = \frac{V_{max}}{\sqrt{R^2 + \left(X_L - X_C\right)^2}} = \frac{V_{max}}{\sqrt{R^2 + \left(\omega_r L - \frac{1}{\omega_r C}\right)^2}}$$

3). Lower cut-off frequency, (f_L)

$$\text{At half power, } \frac{P_m}{2}, \; I = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

$$Z = \sqrt{2}R, \; X = -R \; (\text{capacitive})$$

$$\omega_L = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

4). Upper cut-off frequency, (f_H)

At half power, $\frac{P_m}{2}$, $I = \frac{I_m}{\sqrt{2}} = 0.707 I_m$

$$Z = \sqrt{2}R,\ X = +R\ (\text{inductive})$$

$$\omega_H = +\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

5). Bandwidth, (BW)

$$BW = \frac{f_r}{Q},\quad f_H - f_L,\quad \frac{R}{L}\ (\text{rads})\quad \text{or}\quad \frac{R}{2\pi L}\ (\text{Hz})$$

6). Quality Factor, (Q)

$$Q = \frac{\omega_r L}{R} = \frac{X_L}{R} = \frac{1}{\omega_r CR} = \frac{X_C}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$